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Abstract.

In this paper firm's location decision and government investment plans in less-developed areas are interpreted as a coordination game with and without strategic uncertainty. The latter is distinguished from risk and is modeled by means of the Choquet Expected Utility theory. For the case of no uncertainty, multiple Pareto-ranked equilibria arises. The study of the game under uncertainty shows that the Pareto-dominated equilibrium can be selected in equilibrium for sufficiently pessimistic players, while, if the government is supposed to be optimists, can rationally decide to invest besides any firms' decision.

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Strategic Uncertainty, Choquet Expected Utility, Coordination Games, Local Development.

Introduction.

Location choices of firms have extremely relevant consequences for both firms' profits and the development of a specific area or region. The issue of locating in a less developed area represents a strategic choice for a firm, which has to decide whether to invest in that area and for the "government"[1] as well, which chooses whether to convoy resources for the development of that specific place or divert resources for some alternative programs. Clearly, both problems are interactive, in the sense that the realization of the desired outcome depends as much on the decision of each decision maker as on the choices of the other entities which are involved in the problem. In particular, for a firm it is crucial to know whether its public counterpart will invest in the region in which it has planned to locate, by developing infrastructures, financing directly or indirectly the firm, granting "location premia", and so on. Similarly it is equally fundamental for the public sector administrators to know whether firms will take advantage of public investments in that area, since the desired outcomes (such as reducing the unemployment rate, increasing the local product, etc.) depend on both public and private investments.

This problem can be analyzed as a strategic interaction situation and particularly as a coordination game between firms and public sector. This is an extremely powerful tool that helps to clarify the distinction between each player's individual advantage, that drives each players' strategy, and the overall result which depends on the cooperation that players can establish among them. However, it is not free of any drawbacks. In particular, in this sort of games there are typically two Nash-equilibria that can be Pareto-ranked. Thus, setting a game that represent the problem of investing in a specific less-developed area for firms and "government" we will generally identify two Nash-equilibria, one in which all subjects invest and one in which none of them do it. Clearly, both firms and public sector would prefer the former to the latter, but there is no intrinsic characteristic of the game that helps selecting one of them from the other. In this sense, the predictive power of a game-theoretic analysis of this problem would be rather vague and such an application unsatisfactory.

Such a result is related to the impossibility of capturing, within the "standard" game-theoretic framework (as well as for non-strategic decision making models), the distinction between "risk" and "uncertainty", where the former represents uncertain decision's problems that can be tackled by means of a single prior probability distribution (over outcomes or acts), while the latter refers to situation in which this cannot be the case (See Gilboa – Schmeidler (1989), Schmeidler (1989), Sarin – Wakker (1992)). As a matter of fact, in game-theory models players are assumed to have a unique (subjective or objective) additive prior distribution (over other players' strategies) that represent his/her belief. On the other hand, several situations of strategic interaction (such as the investment problem of firms and "government", for instance) appear to

[1] Throughout the paper we will not distinguish among vertically or horizontally differentiated public power within the public sector and we will generally speak of "government" to identify that public sector body that is responsible for public sector intervention in the economy, without specifying how this intervention takes (or should take) place in reality.

be plagued with uncertainty, which arises because of an imperfect knowledge of the characteristic of the game that players are playing.

Applying the distinction between risk and uncertainty to a game-theoretic framework provides a more general framework to study coordination games such as the firm-government one, that encompasses the "standard" model as a specific sub-case, namely the case of no uncertainty. Moreover, it helps solving the problem of equilibria multiplicity by parametrizing different outcome according to an exogenous parameter that expresses players' uncertainty about the decision they are facing.

In this paper we will study the coordination game that arises when firms have to decide whether to locate in a certain area or somewhere else and the government whether to invest in that area or in some alternative project, framing as a game with uncertainty between a representative firm (or a median one) and the government, where both have a specific attitudes towards uncertainty.

We will proceed as it follows. First we will set a game-theoretic framework to define the firms-government investment relationship, and we will show its multiplicity of equilibria under "standard" game-theoretic hypothesis. Then we will introduce the issue of uncertainty formalized by means of the so-called "capacities", that leads to a preference representation known as Choquet Expected Utility. We will next introduce uncertainty into strategic interactions, and we will apply it to our game. We will show that some frequently observed situations might be interpreted as the equilibrium outcome of rational but "pessimistic" or "optimistic" players. In particular, we will see that the Pareto-dominated no investment equilibrium is the only equilibrium solution of the game for firms and government sufficiently pessimists; on the other hand, if the government is sufficiently optimist, it might happen that it will invest in a certain region even if no firm locates in it. This might provide a rationale for the often observed phenomena of public investment projects that are conducted in some area that do not stimulate investment form private firms; similarly, it might suggest that the underdevelopment of some region might be due to an "uncertainty trap" in which they are stacked, that prevents them from being appealing locations for private firm's besides any government investment. We will end up with some conclusive remarks. Proofs will be collected in the Appendix.

The game without uncertainty.

Suppose there are two players in the game: a representative (or a median) firm (denoted as F) that has to decide whether to undertake a certain investment in a potentially rich but still underdeveloped area, and the government (G), that can finance a development program in that region by undertaking some specific investment plans in that area. Firms perspectives are such as that, if it decides to locate in that area and if the government undertakes its investment programs in it, it benefit from its own investment. On the other hand, if it decides to invest but the government did not, it looses the cost of building its plant in that area, obtaining lower benefits than those it would have had if it had decided not to locate in that area.

For the government, we suppose that it aims at increasing the development of a specific area. It can do so by directly financing infrastructures or it might use its resources in a different plan. If it decides to undertake direct public investments and firms follows it (in the sense that choose to locate in that area) it succeeds in its target. However, if it finances public infrastructures but firms do not locate in that area, it does not succeed in reaching its target and it looses also the possibility of using the employed resources in the different plan.

Let us describe the game more formally. The firm has two different strategies: to locate (that from now onward will be denoted as strategy l) or not to locate (strategy $n\text{-}l$). The government chooses between two strategies: to invest (strategy i) or not to invest ($n\text{-}i$). Denote firm's payoff function as $P^F(x, y)$ and government one as $P^G(y,x)$, where $x \in \{l, n\text{-}l\}$ is a firm's strategy and $y \in \{i, n\text{-}i\}$ is a government strategy. The game can be represented as the following collection of sets:

$$\Gamma = [\{F,G\}; (\{l, n\text{-}l\}, \{i, n\text{-}i\}), (P^F(\cdot), P^G(\cdot))] \qquad (1)$$

Let us denote F's payoffs as: $P^F(l, i) = a$, $P^F(l,n\text{-} i) = g$, $P^F(n\text{-}l, i) = P^F(n\text{-}l, n\text{-}i) = b$. Similarly, G's payoffs are denoted as: $P^G(i, l) = \alpha$, $P^G(i, n\text{-}l) = \gamma$, $P^G(n\text{-}i, l) = P^G(n\text{-}i, n\text{-}l) = \beta$. Clearly, given our assumptions on Payoffs, we can rank them as $a > b > g$, $\alpha > \beta > \gamma$; and represent it in figure 1:

Government

Firm		*i*	*n-i*
	l	$a\ ;\ \alpha$	$g\ ;\ \beta$
	n-l	$b\ ;\ \gamma$	$b\ ;\ \beta$

Figure 1

It is elementary to see that game (1) has two Nash-equilibria (N-E) in pure strategies:

Proposition 1. The game Γ= [{F,G}; ({l, n-l}, {i, n-i}), ($P^F(\cdot)$, $P^G(\cdot)$)] has two Nash-equilibria: {l, i} and {n-l, n-i}.

Proof. in Appendix.

Notice that clearly the first N-E Pareto-dominates the second one. However, if F plays n-l, it is optimal for G to play n-i, and vice versa. Given the simultaneity of the game, there is no reason why such a result should not happen. This is a well-known characteristic of all coordination games, which leaves open the problem of the descriptive power of such a model. In particular, should we expect firms and government to coordinate toward investing in the less-developed area or not?

There are several ways to tackle this problem in the literature. For instance, we might suppose that the game is a repeated one, so that we can ask ourselves if there is an equilibrium which is more likely to happen in a repeated interaction setting. We will follow here a different approach, asking ourselves if there is something missing in the structure of the game that enables us to provide a richer description of the game and that can help us to select among equilibria. Our answer is that there is an important aspect which has been neglected in the game's description, namely uncertainty about other players' choices, where uncertainty is formalized by means of the so-called "capacities".

Modeling Uncertainty Through Capacities.

Assume Ω = {...,ω,...} is a set of "state of nature" and let E = 2^Ω be the set of possible "events". A "capacity" is the following non-additive measure:

Definition (capacity): Assume v: $E \to \Re$; v($\cdot$) is a capacity if:

(i) v($\emptyset$) = 0, v(Ω) = 1 [normalization condition]

(ii) For all A,B $\in$ E such that A $\subseteq$ B $\Rightarrow$ v(A) $\leq$ v(B) [monotonicity]

We say that v is convex if, for all A,B $\in$ E, v(A $\cup$ B) $\geq$ v(A) + v(B) - v(A $\cap$ B). It is concave if the previous inequality is reversed. A capacity v($\cdot$) that is both concave and convex is such that: v(A $\cup$ B) = V(A) + v(B) - v(A $\cap$ B), i.e., it is additive.

Defining a monotonic[2] function f: $\Omega \rightarrow \Re$ and an additive distribution $\pi(\cdot)$ on 2^{Ω}, the integral of $f(\cdot)$ with respect to $\pi(\cdot)$ is the following:

$$\int_{\Omega} f(\omega)\, \mathrm{d}\pi(\Omega) = \int_{0}^{\infty} \pi(\{\omega \in \Omega | f(\omega) \geq t\})\mathrm{d}t + \int_{-\infty}^{0} [\pi(\{\omega \in \Omega | f(\omega) \geq t\}) - 1]\mathrm{d}t \qquad (2)$$

Where the existence of the sum of the two Riemann integrals on the r.h.s. is guaranteed by the monotonicity of the integrand and $t \in \Re$. Savage (1954) has provided the appropriate set of conditions (i.e. axioms) for which it exists a mapping from states to consequences f: $\Omega \rightarrow X$, an utility function u: $X \rightarrow \Re$ and an additive probability distribution $\pi(\cdot)$ on 2^{Ω} such that a Decision Maker's (DM) preferences are represented by the expected utility $\int_{\Omega} \mathrm{u}(\mathrm{f}(\omega))\mathrm{d}\pi$, that can be calculated using equation (2). Choquet (1954) has proved that if we replace $\pi(\cdot)$ with a non-additive measure v($\cdot$) the integral in (2) remains well defined, that is:

$$\int_{\Omega} f(\omega)\, \mathrm{dv}(\omega) = \int_{0}^{\infty} \mathrm{v}(\{\omega \in \Omega | f(\omega) \geq t\})\mathrm{d}t + \int_{-\infty}^{0} [\mathrm{v}(\{\omega \in \Omega | f(\omega) \geq t\}) - \mathrm{v}(\Omega)]\mathrm{d}t \qquad (2')$$

where v(Ω) =1 by normalization. Call (2') Choquet Integral (CI).

The problem of introducing uncertainty in the Theory of Decision can thus be restated as the study of the most appropriate axioms that leads to a representation of preferences with an "expected utility" functional and a non-additive probability measure that can be calculated with CI.

This problem has first been tackled by Gilboa (1987), Schmeidler (1989), Sarin – Wakker (1992). They have shown that replacing Savage's axioms with a (somehow) less-demanding one it is possible to represent DM's preferences through a specific "expected utility" functional which is defined with respect to a capacity and that can be calculated by means of the CI. More formally (Ghirardato – Marinacci, (1999)), say that a DM's preference ordering is a Choquet Expected Utility (CEU) ordering according to the following:

<u>Definition (Choquet Expected Utility preferences' ordering).</u> A preferences' ordering is said to be a CEU ordering if there exists an (underlying) utility function u: $X \rightarrow \Re$, and a unique capacity on the power set of Ω, v($\cdot$), such that for all f, f* $\in$ E (where f denotes an "act" i.e. f:$\Omega \rightarrow X$)

[2] Monotonicity here means that $t' \geq t \Rightarrow \{\omega: f(\omega) \geq t'\} \subseteq \{\omega: f(\omega) \geq t\}$, since we have not defined any specific structure for the set Ω. Usual monotonicity definition follows from the previous one supposing that Ω is an euclidean space.

$$f \succeq f^* \textit{ iff } \int_\Omega (u(f(\omega))dv(\omega) \geq \int_\Omega (u(f^*(\omega))dv(\omega)$$

The integral in the previous definition is called Choquet Expected Utility. It has been proved[3] that (under some conditions) there exist an unique (up to a positive affine transformation) utility function and an unique capacity that represent a CEU preferences' ordering, i.e. that represent a DM's ordering by means of a CEU. Throughout this work we will denote CEU (and EU) by means of capital U, to distinguish it from the "underlying" utility function u. If we suppose that there are *n* states $\{\omega_1, ..., \omega_n\}$ and we rank utility as $u(f(\omega_1)) \geq ... \geq u(f(\omega_n))$, we can write the CEU as:

$$\int_\Omega (u(f(\omega))dv(\omega) = \sum_{k=1}^{n} u(f(\omega_k))[v(\textstyle\bigcup_{j=1}^{k}(\omega_j)) - v(\bigcup_{j=1}^{k-1}(\omega_j))] \tag{3}$$

where we set $v(\omega_0) = 0$ for simplicity of notation.

In order to interpret the previous equations, suppose that there are four possible states only. In this case we would write the CEU as:

$$CEU = u(f(\omega_1))[v(\omega_1)] + u(f(\omega_2))[v(\omega_1 \cup \omega_2) - v(\omega_1)] + u(f(\omega_3))[v(\omega_1 \cup \omega_2 \cup \omega_3) - v(\omega_1 \cup \omega_2)] + u(f(\omega_4))[1 - v(\omega_1 \cup \omega_2 \cup \omega_3)] \tag{4}$$

Consider the case of $v(\cdot)$ strictly convex. See that $[1 - v(\omega_1 \cup \omega_2 \cup \omega_3)] > v(\omega_4)$; $[v(\omega_1 \cup \omega_2 \cup \omega_3) - v(\omega_1 \cup \omega_2)] > v(\omega_3)$; $[v(\omega_1 \cup \omega_2) - v(\omega_1)] > v(\omega_2)$. This means that the DM over-evaluate in her CEU the outcomes that she ranks as worst with respect to the weights she would have adopted if she had expressed her attitude towards uncertainty by means of an additive probability. We will define such a DM as a "pessimist" one, to stress the "overweight" that she attaches to her worst outcomes. Similarly, suppose v is strictly concave. Now $[1 - v(\omega_1 \cup \omega_2 \cup \omega_3)] < v(\omega_4)$; $[v(\omega_1 \cup \omega_2 \cup \omega_3) - v(\omega_1 \cup \omega_2)] < v(\omega_3)$; $[v(\omega_1 \cup \omega_2) - v(\omega_1)] < v(\omega_2)$. The DM would now under-evaluate in her expected utility the states that she conceive as less favorable ones, i.e. would express an optimistic attitude. Thus, we will define such a player as an "optimist".

Finally, before we proceed further, let us introduce here the concept of "degree of uncertainty" which will turn out to be useful to parametrize uncertainty in our game. Following Marinacci (2000) we can characterize capacities by the following vector of parameters:

$$\gamma(v) = \max \{v(A) + v(\Omega \backslash A) \mid A \subset \Omega\} \tag{5}$$

this vector expresses the degree of confidence on each player's assessment, since measures the "distance from additivity" attached to each capacity. To see it more clearly, consider the following capacity, called "simple capacity":

$$v(A) = \gamma\pi(A)\ \forall\ A \subset \Omega, \tag{6}$$

where $\pi(\cdot)$ is an additive probability and $\gamma \in [0,1]$.

It is easy to see that $\gamma(v) = \gamma$. In words, the vector $\gamma(v)$ degenerates to the scalar γ that parametrizes the degree of confidence: $\gamma = 1$ means full confidence, (thus the player has an additive probability) while $\gamma = 0$ expresses "zero confidence" i.e. full ignorance on the situation he/she is facing.

Introducing Uncertainty in Games.

Assume I = {1,2,...,n} is the set of players. Define now with $s_i \in S_i$ a generic pure strategy of player i; S_i is the (finite) set of all players'i pure strategies. $S = \times_{j \in I} S_j$ is the set of outcomes, and $s \in S$ denotes a generic element of S. Define also $S_{-i} = \times_{\substack{j \in I \\ i \neq j}} S_j$ as the set of all others players' strategies except player i ones, and allow throughout the paper to identify with subscript $_{-i}$ all players except player i. Assume u_i represents the utility function of player i, i.e. $u_i : S \rightarrow \Re$, and define a generic normal-form game as $\Gamma = [I, (S_i)_{i \in I}, (u_i)_{i \in I}]$.

Let Σ_i be the set of all (additive) probability distributions on S_i. Let, for a given $\sigma_i \in \Sigma_i$, $\sigma_i(s_i)$, $i \in I$, representing the (additive) probability that σ_i attaches to s_i. Σ_i represents the mixed strategy set of player i. Denote the "mixed extension" of Γ as $\Gamma^* = [I, (\Sigma_i)_{i \in I}, (U_i)_{i \in I}]$, where U_i is the function $U_i : \Sigma \rightarrow \Re$, i.e. $U_i(\sigma) = \Sigma_{s \in S}[\Pi_{i \in I}\, \sigma_i] u_i(s_i, s_{-i})$.

Recall that we can interpret a mixed strategy as a player's belief. In this case, we define a belief of player i (denoted as β_i) as a probability distribution on S_{-i}; i.e., $\beta_i = \times_{j \neq i} \sigma_j$. According to this interpretation, players actually choose pure strategies, a mixed strategy of player i represents the beliefs that <u>all other players</u> maintain about the possibility that player i plays a certain set of her pure strategies, and a N-E in mixed

[3] See Schmeidler (1989).

strategies is a set of equilibrium beliefs which satisfies some specific properties, namely, that each strategy which has a positive probability of being played is "optimal". More formally, σ is a N-E *iff*

$$s_i \in \text{supp}(\sigma_i) \Rightarrow s_i \in \underset{s_i \in S_i}{\arg\max}\ U_i(s_i, \sigma_{-i}),\ \forall\ i \in I \tag{7}$$

where $\text{supp}(\sigma_i) = \{s_i \in S_i \mid \sigma_i(s_i) > 0\}$ denotes the support of a strategy profile σ_i.

In words, for a system of beliefs to be a N-E players' beliefs have to coincide with the mixed strategy which is actually played, or, at least, each pure strategy in the support of mixed strategies of all players must be optimal for that player, i.e., it must be in her best-response.

In this "standard" framework uncertainty plays no role, even though uncertain situations seem likely to arise in a game-theoretical setting. In fact, a player might not know who her opponents are. She might assume, or fear, that her opponents have undergone some previous agreement against her, or, on the contrary, she might believe that they are "altruistic" enough to favor her. More generally, a player might not rely on her opponents' rationality or might suppose that they do not rely on her own rationality, and so on. Furthermore, even if all previous consideration are not believed to be relevant from a theoretical standpoint, there is still the problem of N-E multiplicity, so that, if a N-E is to be interpreted as a stable prescription of play, there is no certainty about which is the effective solution that arise in GT[4].

A possible solution to introduce uncertainty in a game (Dow and Werlang (1994), Marinacci (1996), Eichberer and Kelsey (2000)) is to assume that each player forms his own beliefs about other players' behavior through a capacity, that represents his attitude towards uncertainty. In other words, players play their pure strategies while other players' mixed strategies are interpreted as his non-additive beliefs, and each player maximizes his CEU given such uncertain beliefs[5].

Assume that each player has a capacity $v(\cdot)$ on S_{-i} that we denote as v_i. Such a non-additive distribution on S_{-i} represents the belief of player i about all other players behavior. This belief encompasses the possibility that the environment (i.e. the game) is uncertain and expresses players' i uncertainty aversion. Using the definition of the CEU w.r.t. the non-additive measure $v(\cdot)$, we can define the expected payoff of player's i from playing s_i, holding the non additive beliefs v_i as:

[4] As it is well known, there is a huge literature in GT that deals with the problem of multiplicity of N-E, which goes under the (somehow imprecise) name of "equilibrium refinements". However, there is no solution concept within this literature that deals with players' strategic uncertainty, i.e. uncertainty due to other players strategies' choice (that we are arguing here might arise also because of multiplicity of equilibria).

[5] In other words, it is supposed that the set of "states of nature" Ω coincides with the set of all other players strategies (S_{-i}). This justifies the definition of this type of uncertainty as "strategic".

$$U_i(s_i , v_i) \equiv \int u_i (s_i, s_{-i}) \mathbf{d}v_i, \tag{8}$$

Recall the definition of the support of a mixed strategy s as the smallest set whose complement has measure zero. When we move to a non-additive belief, we need to re-define it since a capacity might be such as to attach measure zero to an event and zero to its complement. This capacity would still influence a player's decision, but would be neglected by the usual definition of a support. To overtake this problem, the support of a capacity can be re-defined as the smallest set whose complement has measure zero, i.e.:

$$E \subseteq S_{-i} \text{ is a supp(v) if } v(E^c) = 0 \text{ and } F \subset E \Rightarrow v(F^c)>0. \tag{9}$$

We are now ready to define the "uncertainty extension" of a game. Let $\{I^p\}$, $\{I^o\}$ be a partition of I. $\{I^p\}$, $\{I^o\}$ are the sets of pessimists ands optimists player respectively, that is to say, the set of players who maximize their own CEU w.r.t. a convex capacity or a concave one, that represent pessimists and optimists' beliefs respectively. More formally:

$$U_i(s_i , v_i) = \int u_i (s_i, s_{-i})\mathbf{d}v_i \equiv U^p_i(s_i , v_i), \text{ if } v_i \text{ is convex;} \tag{10}$$

$$U_i(s_i , v_i) = \int u_i (s_i, s_{-i})\mathbf{d}v_i \equiv U^o_i(s_i , v_i), \text{ if } v_i \text{ is concave.} \tag{11}$$

Consider the following definition:

<u>Definition 3.4[6]:</u> $\Gamma^{\upsilon g} = [(I = \{I^p\}\cup\{I^o\}), (S_i)_{i\in I}, (U_i)_{i\in I^p}, (U_i)_{i\in I^o}]$ is a "Generalized Game with Uncertainty", where $(U_i)_{i\in I^p}$, as it has been defined in (10), is the CEU of a pessimistic player and $(U_i)_{i\in I^o}$, defined in (11), is the CEU of an optimist.

we can now extend the N-E solution concept defining an Equilibrium with Uncertainty for a Generalized game, or (briefly) a "Generalized Equilibrium with Uncertainty" (GE-U) as:

[6] Marinacci (1996) has constructed a similar concept, called "generalized ambiguous games" for the case of i = {1,2} only.

Definition 3.5[7] (Generalized Equilibrium with Uncertainty). a set of capacities $v = \{v_1,...,v_n\}$ is a Generalized Equilibrium with Uncertainty for the game Γ^{ug} if there exists $supp(v_i)$ such that:

$$supp(v_i) \subseteq \underset{j\in I\setminus\{i\}}{\times} B_j(v_j),\ \forall\ i \in \{I^p\}\cup\{I^o\}$$

where $B_j(v_j)$ is defined as:

$$B_j(v_j) = \underset{s_i\in S_i}{\arg\max} \begin{cases} U_i^p(s_i, v_i) & if\ i \in I^p \\ U_i^o(s_i, v_i) & if\ i \in I^o \end{cases}$$

and $U_i^p(\cdot)$, $U_i^o(\cdot)$ have been defined in (10), (11).

The Game with Uncertainty.

Let us now consider the investment-location game as a generalized game with uncertainty, and consequently its equilibria as Generalized Equilibria with Uncertainty. Assume that v_F represents the belief of *F*, that places the weight q_i to strategy *i* of *G* and the weight q_{n-i} to strategy *n-i*; similarly, v_G represents the belief of *G*, that attaches the weight q_l to strategy *l* of *F* and the weight q_{n-l} to strategy *n-l*.

Consider first the case of *F* and *G* both pessimists. Calculating *F*'s CEU, we have:

$$U^p{}_1(s_1, v_1) \equiv P^F{}_{CEU}\ (l, \cdot\ ) = a\cdot(q_i) + g\cdot(1-q_i); \tag{12}$$

$$U^p{}_1(s_2, v_1) \equiv P^F{}_{CEU}\ (n-l, \cdot\ ) = b \tag{13}$$

Similarly, calculating G's CEU, we obtain:

$$U^p{}_2(s_1, v_1) \equiv P^G{}_{CEU}\ (i, \cdot\ ) = \alpha\cdot(q_l) + \gamma\cdot(1-q_l); \tag{14}$$

$$U^p{}_2(s_2, v_1) \equiv P^G{}_{CEU}\ (n-i, \cdot\ ) = \beta \tag{15}$$

We can now easily prove the following:

[7] Eichberger - Kelsey (2000) have proposed a similar concept, called Equilibrium with Uncertainty, for the case of pessimistic players only.

Proposition 2. Strategies profiles {i, l} and {n-i, n-l} can be sustained as GE-U of the game:

$\Gamma^{\upsilon g}$= [{F,G}; ({l, n-l}, {i, n-i}), ($P^F_{CEU}(\cdot)$, $P^G_{CEU}(\cdot)$)]

Proof. In appendix.

Proposition 2 provides the analytical counterpart for the case of uncertainty of Proposition 1. However, if the game is such as to encompasses a sufficiently high degree of uncertainty, pessimistic players will play only the "safe" strategy *n-l* and *n-i*, thus ruling out the Pareto-dominating "cooperative" equilibrium. To see it, let us suppose that each player's belief is "simple", i.e. it is of the form:

$q_j = \gamma_F\pi_j$, j = i, n-i, for F. $q_k = \gamma_G\pi_k$, k = l, n-l, for G (16)

Recall that in this case γ_i expresses the degree of confidence of player i = F, G. Consider the following two Lemma:

Lemma 1: If $\gamma_F < (b\text{-}g)/(a\text{-}g)$, {$n$-$l$} is the only strategy that can be played in equilibrium by F.

Proof. In Appendix.

Lemma 2: If $\gamma_G < (\beta-\gamma)/(\alpha-\gamma)$, {$n$-$i$} is the only strategy that can be played in equilibrium by G.

Proof. In Appendix.

Then the following Proposition holds true:

Proposition 3: In the game with uncertainty $\Gamma^{\upsilon g}$= [{F,G}; ({l, n-l}, {i, n-i}), ($P^F_{CEU}(\cdot)$, $P^G_{CEU}(\cdot)$)], with F and G both pessimists, there exist a threshold level of uncertainty for F, $\gamma^*_F = (b\text{-}g)/(a\text{-}g)$ and a threshold level of uncertainty for G, $\gamma^*_G = (\beta-\gamma)/(\alpha-\gamma)$, below which only the strategy profile {n-i, n-l} can be supported in a GE-U.

Proof. In Appendix.

Proposition 3 provides an interesting rationale to justify the selection of the Pareto-dominated no-investment equilibrium. The problem that both firms and government do not invest even if they would mutually benefit from it might be explained because of their pessimistic attitude towards uncertainty, coupled with an extremely low degree of confidence about their investment problem.

Suppose now that the government is optimist, while the firm is still pessimist[8]. Calculating now G's CEU, we have:

$$U^{o}_{2}(s_1, v_1) \equiv P^{G}_{CEU}(i, \cdot) = \alpha \cdot (1 - q_{n\text{-}l}) + \gamma \cdot (q_l); \tag{17}$$

$$U^{p}_{2}(s_2, v_1) \equiv P^{G}_{CEU}(n\text{-}i, \cdot) = \beta \tag{18}$$

By replicating the proof of Proposition 1 it is straightforward to show that both strategy profiles {*l*, *i*}, {*n-l*, *n-i*} can be supported in a GE-U However, in this case, a third GE-U arises in which *F* plays *n-l* and G *i*:

Proposition 4. The strategy profile {*n-l*, *i*} can be supported in a GE-U of the game $\Gamma^{\upsilon g}= [\{F,G\}; (\{l, n\text{-}l\}, \{i, n\text{-}i\}), (P^{F}_{CEU}(\cdot), P^{G}_{CEU}(\cdot))]$, when *G* is optimist and *F* pessimist.

Proof. in Appendix.

This Proposition suggest an interesting explanation for all those cases in which public investments are not followed by private ones. Under the approach we have been following here such an outcome should not be interpreted as an irrational move of a government which is investing in an underdeveloped area in which firms are not willing to locate, nor should be seen as a wrong decision undertaken by firms that do not take advantage of public investments. In fact, it would not indicate a problem of coordination failure, but rather it would signal a specific uncertainty attitude of firms and "governments", namely pessimism and optimism respectively.

Conclusion.

[8] For instance, a government might be following an "optimistic" electoral program, under-evaluating the negative consequences of its activity, while this seems a less plausible assumption for the firm(s) (management). In other words,

In this paper we have analyzed the game that arises when a firm has to decide whether to locate in an area that needs public investment and the government can choose whether to invest in the development of that area or not. The game is a coordination one, since the outcome depends on the possibility of undertaking the "same" strategy by both types of agent. First of all, we have framed it as a "standard" game, namely, as a strategic interaction situation in which players do not exhibit uncertainty about other players' choices. For this game, a multiplicity of equilibria arises that can be Pareto-ranked, but nothing more can be said about it. In particular, there is no clear sign about the effective strategy plan that is going to be played. Taking into account the distinction between risk and uncertainty a more rich analysis can be implemented. In order to do so, we have first presented the Choquet Expected Utility model, in which attitude towards uncertainty is captured by means of the so-called capacities, i.e. non-additive measures. Then, we have introduced uncertainty in the "standard" game-theoretical setting, defining the concept of a "generalized game with uncertainty" and the "generalized equilibrium with uncertainty". The latter, in particular, can be seen as a generalization of the Nash-Equilibrium concept since it encompasses the possibility that players exhibit pessimism or optimism about the decision problem that are facing, represented through convex or concave capacities, respectively. We have then applied his new frame to our private location-public investment problem, showing that a broader set of behaviors can be interpreted as the rational, i.e. utility maximizing, behavior of pessimistic or optimistic payers. In particular, if both the government and the (representative) firm are sufficiently pessimists, only the Pareto-ranked equilibrium arises an the equilibrium play of the game (where "sufficiently" means that players' degree of confidence falls below a certainty threshold).

The degree of confidence γ is an exogenous parameter of the game, that depends on players' feeling about the situation they are facing. It seems plausible to assume that it is a function of some "external" characteristic of the environment. For instance, it might depends on some objective figures related to the level of danger of a certain area, its "social stability", etc. Under this hypothesis our analysis shows that the problems of development of a specific area depends crucially on all those aspect that might foster uncertainty: an highly dangerous place would be stacked in an "uncertainty trap" since firms would never choose to locate in it, even if the government is investing in infrastructures in that area. A better politics, in this case, would be to address public resources toward the reduction of the uncertainty, that is to say, to promote "safety". On the other hand, the model provides also a theoretical explanation for the often observed problem of public investments that are not able to stimulate growth of a private activity. In the mark of the "standard" analysis this phenomenon might be due to player's inability to converge to the Nash-equilibrium, or as a failure of the "rationality" hypothesis over which game-theoretical analyses are based. That is to say, real players might not been able to evaluate their best-response or might not be utility

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the idea is that the "implicit" cost of being optimist (and going wrong) is less stringent in the political market than in the commodities' one.

maximizers at all. In both cases, there is a discrepancy between the theoretically correct behavior and the effective one.

Our study provides a different explanation for such an outcome that needs not to recall any dichotomy between theoretical prescriptions and actual plays. In fact, Proposition 5 shows that this might be the rational play of (Choquet Expected) Utility maximizers players. In particular, the play "not to locate" for firm, "invest" for the government is a Generalized equilibrium with uncertainty for the game with uncertainty in the case of an optimistic government and a pessimistic firm.

Clearly, our analysis has been rather introductory and simple. We have not considered the issue of how in reality firms investment's plans are undertaken, nor the possibility of collusion among firms and public bodies. Moreover, we have considered static games only, that neglect the possibility of learning over time.

Nevertheless, it has been an attempt to take a new and highly technical tool, such as the Choquet Expected Utility theory, to the ground of effective economic problem. We believe that success of a new theory depends on its ability to interpret effective situations as much as on its coherence and theoretic depth. This has been a move towards this direction; further analysis will show if it has been worth-while.

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Appendix.

Proof of Proposition 1. Given payoffs' structure, *l* is a best-response to *i* and *i* is a best-response to *l*, thus {*l*, *i*} is a N-E of the game; similarly for {*n-l*, *n-i*} since *n-l* is a best-response to *n-i* and *n-i* is a best response to *n-l*. Q.E.D.

Proof of Proposition 2. See that $\arg\max_x P^F_{CEU}(\cdot, v_F) = \{l\}$ *iff* $q_i > (b-g)/(a-g)$; $\arg\max_x P^F_{CEU}(\cdot, v_F) = \{n\text{-}l\}$ *iff* $q_i < (b-g)/(a-g)$; $\arg\max_y P^G_{CEU}(\cdot, v_G) = \{i\}$ *iff* $q_l > (\beta-\gamma)/(\alpha-\gamma)$; $\arg\max_y P^G_{CEU}(\cdot, v_G) = \{n\text{-}i\}$ *iff* $q_l < (\beta-\gamma)/(\alpha-\gamma)$. Consider the following two systems of beliefs:

(i) $v_F' = (q_i, q_{n-i}) = (w,0)$, $w > (b-g)/(a-g)$; $v_G' = (q_l, q_{n-l}) = (z,0)$, $z > (\beta-\gamma)/(\alpha-\gamma)$; this is a GE-U, since $supp(v_1') = \{t_1\}$, $supp(v_2^{*\prime}) = \{s_1\}$; $\arg\max_x P^F_{CEU}(\cdot, v_F) = \{l\}$, $\arg\max_y P^G_{CEU}(\cdot, v_G) = \{i\}$. In this GE-U, the corresponding equilibrium strategies are{*l*} for *F* and {*i*} for *G*.

(ii) $v_1'' = (q_{t1}, q_{t2}) = (0, w)$, $w > 0$ and $v_2'' = (q_{s1}, q_{s2}) = (0, z)$, $z > 0$; This is GE-U. In fact, $supp(v_1'') = \{t_2\}$, $supp(v_2'') = \{s_2\}$; $B_1(v_1'') = \{s_2\}$, $B_2(v_2'') = \{t_2\}$. The corresponding equilibrium strategies are {*n-l*} for *F* and {*n-i*} for *G*. Q.E.D.

Proof of Lemma 1. See that $\gamma_F < (b-g)/(a-g) \Rightarrow q_i < \forall\, \pi_i \in [0,1]$. Thus $\arg\max_x P^F_{CEU}(\cdot, v_F) = \{n\text{-}l\}$ is the only best-response for *F*. Q.E.D.

Proof of Lemma 2. See that $\gamma_G < (\beta-\gamma)/(\alpha-\gamma) \Rightarrow q_l < (\beta-\gamma)/(\alpha-\gamma)\ \forall\, \pi_l \in [0,1]$. Thus $\arg\max_y P^G_{CEU}(\cdot, v_G) = \{n\text{-}i\}$ is the only best-response for *G*. Q.E.D.

Proof of Proposition 3. Given our payoff's structure, $(b-g)/(a-g) \in (0,1)$, $(\beta-\gamma)/(\alpha-\gamma) \in (0,1)$. This and Lemma 1, 2 prove the Proposition. Q.E.D.

Proof of Proposition 4. *F*'s best-responses are unchanged w.r.t. Proposition's 2 ones: $\arg\max_x P^F_{CEU}(\cdot, v_F) = \{l\}$ *iff* $q_i > (b-g)/(a-g)$; $\arg\max_x P^F_{CEU}(\cdot, v_F) = \{n\text{-}l\}$ *iff* $q_i < (b-g)/(a-g)$. Calculating G's best response, we have now: $\arg\max_{y}$ P^{G}_{CEU} $(\cdot, v_G) = \{i\}$ *iff* $q_{n\text{-}l} < (\alpha-\beta)/(\alpha-\gamma)$; $\arg\max_{y}$ P^{G}_{CEU} $(\cdot, v_G) = \{n\text{-}i\}$ *iff* $q_{n\text{-}l} > (\alpha-\beta)/(\alpha-\gamma)$. Consider the following system of beliefs:

$v_1'' = (q_i, q_{n\text{-}i}) = (w,0)$, $w \in (0,(b\text{-}g)/(a\text{-}g))$, $v_2'' = (q_l, q_{n\text{-}l}) = (0, z)$, $z \in (0, (\alpha-\beta)/(\alpha-\gamma))$; we have that $supp(v_2'') = \{n\text{-}l\} \subseteq B_1(v_1'')$, $supp(v_1'') = \{i\} \subseteq B_2(v_2'')$; Thus this is a GE-U. Q.E.D.